



Investigational Allowances

Relationship to Magruder % RSDs

Caution: There will be math.

Investigational Allowance aka IA

- The IA is calculated to statistically represent 99% of the analytical results from labs running the same analyte in the same sample.
 - NPK IA's follow a specified Table in the OP.
 - Other guaranteed mineral nutrient IA's follow specified linear equations, also in the OP.
 - The IA is normally subtracted from a lab result to indicate a minimum allowed on the Guarantee.
 - Here we wish to compare IA with the current status of inter laboratory reproducibility Robust SDs seen in recent Magruder data.
-

Investigational Allowance aka IA

- IA represents 99% of the measurements (R. C. Rund, OP # 28, 1975).
- From Rund, **IA/2.33** represents 68% of the measurements under a Normal curve or 1 standard deviation.
- From the OP IA Tables we calculate an **IA at the Rob. Mean**. Then convert it to the IA_{SD} representing 68% of the measurements or 1 standard deviation.

$$IA_{SD} = \frac{IA_{Rob. Mean}}{2.33}$$

In the following I refer to IA_{SD} as IA.

This allows us to compare IA to Rob. SD

Investigational Allowance aka IA

The IA Ratio

$$\text{IA Ratio} = \frac{\text{SD}_{\text{Robust}}}{\text{IA}_{\text{SD}}}$$

In other words: How many IAs in the Robust SD (between labs)?

I went back and calculated IA ratios for 2017 samples and included them with 2018 report data.

Analyte IA Ratios 2017 to 06/2018

Sample #	Sample Name	n	Rob. Mean	Rob. SD	IA from OP	IA Ratio
Total Nitrogen (%)		ORDERED				
180511	Grade 46-0-0	87	46	0.468	0.880	1.24
170311	Grade 28-0-0 Liquid SR	77	28	0.352	0.835	0.98
170211	Grade 27-0-0	68	27	0.849	0.816	2.42
170811	Grade 21-0-5	82	21	0.307	0.740	0.97
170411	Grade 15-15-15 + Micros	81	20	0.405	0.729	1.29
170111	Grade 18-46-0 DAP	67	18	0.268	0.704	0.89
170511	Grade 16-1-0	78	17	0.217	0.683	0.74
180211	Grade 14-14-14	76	14	0.212	0.628	0.79
170911	MAP plus S, 12-40-0	73	12	0.212	0.615	0.80
171111	Grade 12-0-0-26S	70	12	0.231	0.611	0.88
180411	Grade 12-24-12	86	12	0.230	0.607	0.88
170711	Grade 7-25-40	67	7	0.528	0.535	2.30
171211	Grade 5-15-30	67	5	0.144	0.511	0.66
180311	Grade 3-1-2	75	3	0.130	0.490	0.62
170611	Grade 2-15-15 w/PO3	82	2	0.150	0.490	0.71
180111	Secondary & Micronutrients	10	0.6	0.046	0.490	0.22
180512	TSP + Zn	8	0.1	0.035	0.490	0.17
Direct Available Phosphorus as P2O5 (%)						
170111	Grade 18-46-0 DAP	38	47	0.727	1.008	1.68
171012	TSP, 0-45-0	57	46	0.781	0.994	1.83
180512	TSP + Zn	57	45	0.584	0.988	1.38
170911	MAP plus S, 12-40-0	54	39	0.828	0.887	2.17
180411	Grade 12-24-12	57	25	0.724	0.730	2.31
170711	Grade 7-25-40	48	25	0.535	0.730	1.71
170411	Grade 15-15-15 + Micros	47	18	0.634	0.709	2.08
171211	Grade 5-15-30	47	15	0.301	0.700	1.00
180211	Grade 14-14-14	54	14	0.340	0.700	1.13
180311	Grade 3-1-2	39	3	0.525	0.670	1.82
170511	Grade 16-1-0	46	1	0.115	0.670	0.40

IA ratio = Rob. SD/IA_{SD} Red indicates IA Ratio significantly > 1

Analyte IA Ratios 2017 to 06/2018

Sample #	Sample Name	n	Rob. Mean	Rob. SD	IA from OP	IA Ratio
Soluble Potassium as K2O (%)			ORDERED			
170711	Grade 7-25-40	85	40	1.040	1.545	1.57
171211	Grade 5-15-30	80	30	0.730	1.402	1.21
170611	Grade 2-15-15 w/PO3	87	17	0.817	0.992	1.92
180211	Grade 14-14-14	92	15	0.531	0.889	1.39
180411	Grade 12-24-12	97	13	0.562	0.826	1.58
170811	Grade 21-0-5	91	5	0.188	0.434	1.01
180311	Grade 3-1-2	80	4	0.255	0.410	1.45
170411	Grade 15-15-15 + Micros	77	2	0.220	0.410	1.25
170511	Grade 16-1-0	21	2	0.082	0.410	0.47
180512	TSP + Zn	13	0.2	0.114	0.410	0.65
171012	TSP, 0-45-0	8	0.2	0.095	0.410	0.54
170111	Grade 18-46-0 DAP	11	0.2	0.076	0.410	0.43
170911	MAP plus S, 12-40-0	12	0.1	0.031	0.410	0.18
180111	Secondary & Micronutrients	9	0.1	0.025	0.410	0.14
170211	Grade 27-0-0	6	0.1	0.073	0.410	0.41
Acid Soluble Calcium (%)						
180111	Secondary & Micronutrients	63	19	1.216	1.000	2.83
180512	TSP + Zn	56	14	0.907	0.915	2.31
171012	TSP, 0-45-0	63	13	0.713	0.874	1.90
170811	Grade 21-0-5	17	7	0.487	0.536	2.11
170211	Grade 27-0-0	56	5	0.242	0.425	1.33
180411	Grade 12-24-12	19	4	0.317	0.417	1.77
170511	Grade 16-1-0	17	0.5	0.041	0.225	0.42
170911	MAP plus S, 12-40-0	17	0.4	0.045	0.221	0.47
171211	Grade 5-15-30	44	0.4	0.066	0.219	0.70
170411	Grade 15-15-15 + Micros	23	0.3	0.053	0.217	0.57
180211	Grade 14-14-14	13	0.3	0.045	0.215	0.48
170111	Grade 18-46-0 DAP	14	0.3	0.054	0.214	0.59
180311	Grade 3-1-2	13	0.05	0.014	0.202	0.16
171011	Epsom Salts	10	0.03	0.009	0.201	0.11
170711	Grade 7-25-40	6	0.02	0.021	0.201	0.24

Analyte IA Ratios 2017 to 06/2018

Sample #	Sample Name	n	Rob. Mean	Rob. SD	IA	IA Ratio
Acid Soluble Magnesium (%)		ORDERED				
171011	Epsom Salts	68	10	0.586	0.697	1.96
170411	Grade 15-15-15 + Micros	59	3	0.191	0.336	1.32
170811	Grade 21-0-5	21	3	0.189	0.331	1.33
170211	Grade 27-0-0	55	3	0.193	0.330	1.36
180411	Grade 12-24-12	67	1	0.131	0.273	1.12
170911	MAP plus S, 12-40-0	20	1	0.123	0.267	1.07
171211	Grade 5-15-30	54	1	0.096	0.259	0.86
171012	TSP, 0-45-0	19	1	0.068	0.244	0.65
180512	TSP + Zn	19	1	0.086	0.237	0.85
170111	Grade 18-46-0 DAP	17	1	0.059	0.236	0.59
180211	Grade 14-14-14	48	1	0.052	0.229	0.53
180311	Grade 3-1-2	19	0.4	0.039	0.219	0.41
170511	Grade 16-1-0	18	0.2	0.018	0.210	0.19
180111	Secondary & Micronutrients	15	0.1	0.010	0.206	0.12
170711	Grade 7-25-40	7	0.004	0.005	0.200	0.05
170611	Grade 2-15-15 w/PO3	8	0.001	0.001	0.200	0.01
Total Sulfur (%)						
171111	Grade 12-0-0-26S	38	26	0.832	1.000	1.94
170511	Grade 16-1-0	41	19	0.789	1.000	1.84
180111	Secondary & Micronutrients	49	16	0.897	1.000	2.09
171011	Epsom Salts	51	13	0.565	0.859	1.53
180211	Grade 14-14-14	45	10	0.401	0.724	1.29
170911	MAP plus S, 12-40-0	47	7	0.362	0.551	1.53
171211	Grade 5-15-30	37	5	0.868	0.473	4.27
170811	Grade 21-0-5	44	5	0.240	0.467	1.20
170411	Grade 15-15-15 + Micros	45	5	0.496	0.429	2.69
170111	Grade 18-46-0 DAP	10	2	0.084	0.310	0.63
180411	Grade 12-24-12	39	1	0.111	0.258	1.00
171012	TSP, 0-45-0	8	1	0.092	0.256	0.83
180311	Grade 3-1-2	10	1	0.066	0.247	0.62
180512	TSP + Zn	7	1	0.099	0.240	0.96
170211	Grade 27-0-0	8	0.2	0.068	0.211	0.76
170711	Grade 7-25-40	6	0.01	0.007	0.201	0.09

Analyte IA Ratios 2017 to 06/2018

Sample #	Sample Name	n	Rob. Mean	Rob. SD	IA from OP	IA Ratio
Acid Soluble Boron (%)			ORDERED			
180111	Secondary & Micronutrients	52	0.5	0.042	0.079	1.23
170411	Grade 15-15-15 + Micros	37	0.3	0.040	0.045	2.10
171211	Grade 5-15-30	41	0.2	0.027	0.040	1.57
180211	Grade 14-14-14	47	0.2	0.016	0.034	1.13
180411	Grade 12-24-12	47	0.06	0.014	0.012	2.73
170911	MAP plus S, 12-40-0	12	0.01	0.014	0.005	6.28
170111	Grade 18-46-0 DAP	10	0.01	0.007	0.005	3.24
180512	TSP + Zn	8	0.01	0.005	0.004	3.00
170511	Grade 16-1-0	7	0.004	0.005	0.004	3.25
170811	Grade 21-0-5	7	0.004	0.004	0.004	2.85
180311	Grade 3-1-2	6	0.002	0.003	0.003	1.89
Acid Soluble Cobalt (ppm)						
180211	Grade 14-14-14	21	376	26.116	113.746	0.53
171211	Grade 5-15-30	18	288	40.540	87.285	1.08
170411	Grade 15-15-15 + Micros	21	32	6.663	10.708	1.45
170811	Grade 21-0-5	22	13	3.344	4.842	1.61
170511	Grade 16-1-0	16	7	1.904	3.154	1.41
171012	TSP, 0-45-0	16	6	1.221	2.924	0.97
180411	Grade 12-24-12	23	4	1.398	2.109	1.54
180512	TSP + Zn	15	3	0.871	1.967	1.03
180111	Secondary & Micronutrients	23	2	0.666	1.560	0.99
170111	Grade 18-46-0 DAP	11	1	0.639	1.368	1.09
170911	MAP plus S, 12-40-0	10	1	0.558	1.354	0.96
170211	Grade 27-0-0	8	1	0.465	1.268	0.85
180311	Grade 3-1-2	14	0.4	0.137	1.124	0.28
170711	Grade 7-25-40	10	0.4	0.302	1.107	0.64
171011	Epsom Salts	9	0.3	0.188	1.101	0.40

IA ratio = Rob. SD/IA_{SD} **Red** indicates IA Ratio significantly > 1

Analyte IA Ratios 2017 to 06/2018

Sample #	Sample Name	n	Rob. Mean	Rob. SD	IA from OP	IA Ratio
Acid Soluble Copper (%)			ORDERED			
170411	Grade 15-15-15 + Micros	57	0.6	0.043	0.066	1.52
180211	Grade 14-14-14	52	0.1	0.017	0.018	2.28
171211	Grade 5-15-30	39	0.03	0.004	0.008	1.07
170811	Grade 21-0-5	34	0.01	0.001	0.006	0.44
171012	TSP, 0-45-0	37	0.01	0.002	0.006	0.63
170511	Grade 16-1-0	29	0.01	0.001	0.006	0.55
180512	TSP + Zn	34	0.01	0.001	0.006	0.55
170911	MAP plus S, 12-40-0	11	0.002	0.002	0.005	1.03
180111	Secondary & Micronutrients	19	0.001	0.001	0.005	0.68
180411	Grade 12-24-12	20	0.001	0.001	0.005	0.33
180311	Grade 3-1-2	21	0.001	0.000	0.005	0.21
170111	Grade 18-46-0 DAP	10	0.001	0.001	0.005	0.30
171011	Epsom Salts	11	0.001	0.000	0.005	0.23
170211	Grade 27-0-0	10	0.0005	0.000	0.005	0.19
170711	Grade 7-25-40	15	0.0003	0.000	0.005	0.06
Acid Soluble Iron (%)						
170511	Grade 16-1-0	57	3	0.219	0.263	1.94
170411	Grade 15-15-15 + Micros	56	2	0.297	0.248	2.79
170811	Grade 21-0-5	63	1	0.125	0.115	2.54
171012	TSP, 0-45-0	20	1	0.041	0.084	1.13
180512	TSP + Zn	22	1	0.054	0.080	1.58
180211	Grade 14-14-14	56	1	0.105	0.068	3.59
180411	Grade 12-24-12	25	1	0.030	0.068	1.04
171211	Grade 5-15-30	49	1	0.046	0.060	1.78
170111	Grade 18-46-0 DAP	18	0.4	0.029	0.048	1.41
170911	MAP plus S, 12-40-0	21	0.3	0.017	0.030	1.31
180111	Secondary & Micronutrients	15	0.1	0.012	0.012	2.19
170211	Grade 27-0-0	16	0.05	0.004	0.010	0.91
180311	Grade 3-1-2	19	0.04	0.009	0.009	2.41
170711	Grade 7-25-40	10	0.01	0.003	0.006	1.26
171011	Epsom Salts	7	0.002	0.001	0.005	0.58
170611	Grade 2-15-15 w/PO3	22	0.001	0.001	0.005	0.45

Analyte IA Ratios 2017 to 06/2018

Sample #	Sample Name	n	Rob. Mean	Rob. SD	IA from OP	IA Ratio
Acid Soluble Manganese (%)			ORDERED			
170411	Grade 15-15-15 + Micros	58	1	0.087	0.127	1.59
180211	Grade 14-14-14	49	0.2	0.012	0.021	1.32
170811	Grade 21-0-5	20	0.04	0.003	0.009	0.92
171211	Grade 5-15-30	41	0.03	0.003	0.008	1.02
171012	TSP, 0-45-0	18	0.02	0.002	0.007	0.75
180411	Grade 12-24-12	23	0.02	0.002	0.007	0.57
170911	MAP plus S, 12-40-0	20	0.02	0.002	0.007	0.71
180512	TSP + Zn	18	0.01	0.004	0.006	1.35
170511	Grade 16-1-0	18	0.01	0.001	0.006	0.57
180111	Secondary & Micronutrients	11	0.01	0.001	0.006	0.36
170111	Grade 18-46-0 DAP	12	0.005	0.000	0.005	0.21
170211	Grade 27-0-0	13	0.005	0.000	0.005	0.16
180311	Grade 3-1-2	13	0.002	0.000	0.005	0.08
171011	Epsom Salts	10	0.002	0.001	0.005	0.30
Acid Soluble Molybdenum (ppm)						
170411	Grade 15-15-15 + Micros	24	1292	238.341	388.515	1.43
180211	Grade 14-14-14	27	826	115.407	248.944	1.08
171211	Grade 5-15-30	25	618	103.623	186.377	1.30
180512	TSP + Zn	23	17	3.684	5.971	1.44
171012	TSP, 0-45-0	23	12	2.513	4.729	1.24
170811	Grade 21-0-5	24	10	2.515	3.910	1.50
180411	Grade 12-24-12	25	8	1.938	3.276	1.38
170111	Grade 18-46-0 DAP	17	4	1.555	2.116	1.71
170511	Grade 16-1-0	11	4	1.042	2.090	1.16
170911	MAP plus S, 12-40-0	17	2	1.049	1.732	1.41
180111	Secondary & Micronutrients	19	2	1.441	1.685	1.99
170211	Grade 27-0-0	11	2	0.470	1.546	0.71
180311	Grade 3-1-2	18	1	0.310	1.399	0.52
171011	Epsom Salts	18	1	0.350	1.398	0.58
170711	Grade 7-25-40	11	1	0.284	1.208	0.55

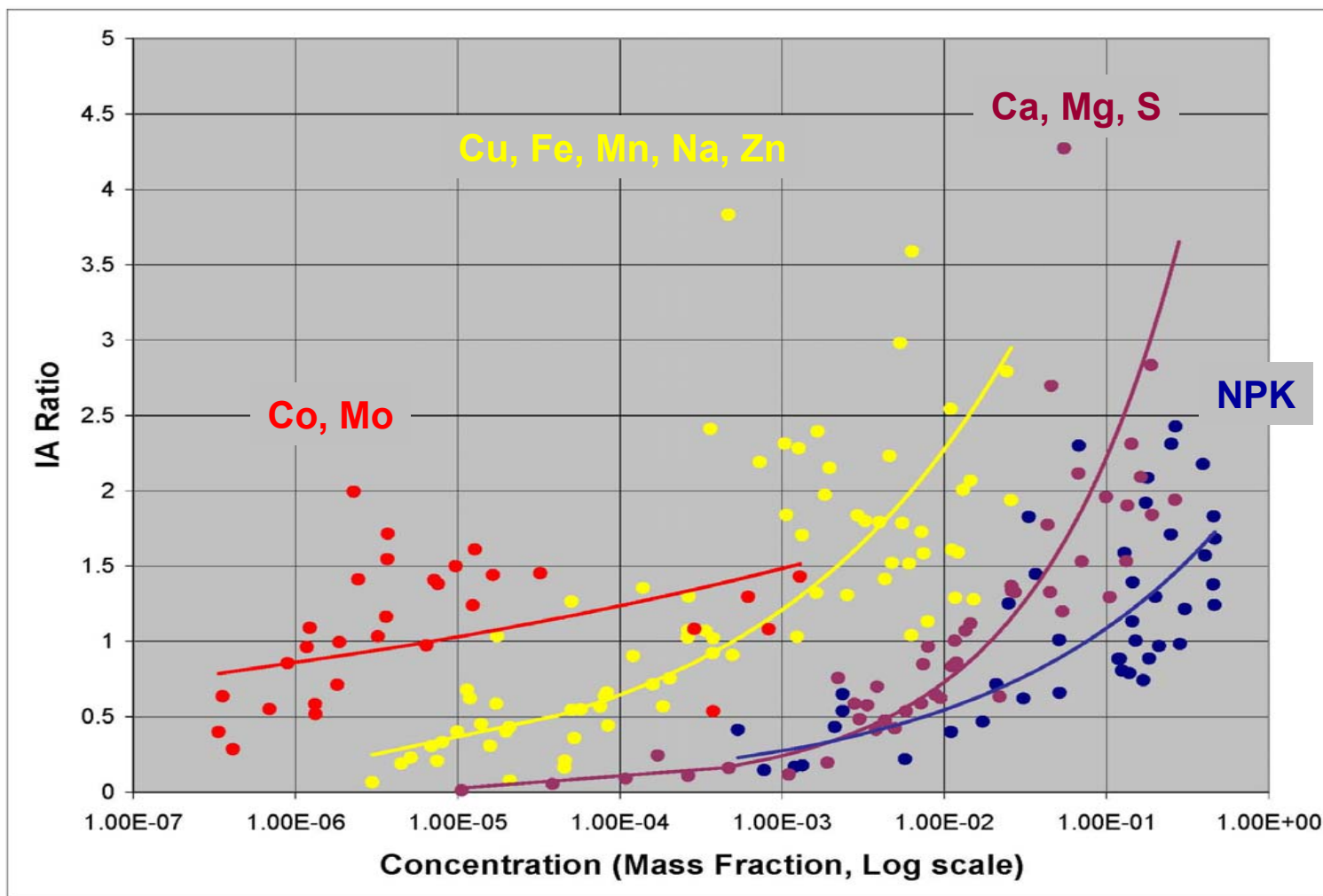
Analyte IA Ratios 2017 to 06/2018

Sample #	Sample Name	n	Rob. Mean	Rob. SD	IA from OP	IA Ratio
Sodium (%)						
ORDERED						
180311	Grade 3-1-2	9	1	0.080	0.116	1.61
171211	Grade 5-15-30	9	1	0.057	0.078	1.73
170411	Grade 15-15-15 + Micros	8	1	0.075	0.058	2.98
180211	Grade 14-14-14	12	0.5	0.049	0.051	2.23
180411	Grade 12-24-12	11	0.4	0.035	0.045	1.79
180111	Secondary & Micronutrients	9	0.3	0.029	0.038	1.80
170711	Grade 7-25-40	6	0.3	0.027	0.034	1.84
170111	Grade 18-46-0 DAP	7	0.2	0.023	0.025	2.15
170211	Grade 27-0-0	8	0.2	0.022	0.022	2.39
170911	MAP plus S, 12-40-0	8	0.1	0.013	0.018	1.70
170511	Grade 16-1-0	8	0.1	0.008	0.017	1.03
180512	TSP + Zn	7	0.1	0.012	0.016	1.84
170811	Grade 21-0-5	10	0.05	0.016	0.010	3.83
Acid Soluble Zinc (%)						
180111	Secondary & Micronutrients	63	2	0.086	0.157	1.28
170411	Grade 15-15-15 + Micros	59	1	0.133	0.150	2.06
170911	MAP plus S, 12-40-0	70	1	0.116	0.135	2.00
171211	Grade 5-15-30	57	1	0.067	0.122	1.29
180512	TSP + Zn	62	0.5	0.034	0.053	1.52
180211	Grade 14-14-14	55	0.2	0.020	0.023	1.97
171012	TSP, 0-45-0	38	0.1	0.015	0.015	2.31
170111	Grade 18-46-0 DAP	31	0.04	0.004	0.009	1.02
170811	Grade 21-0-5	36	0.03	0.004	0.008	1.07
180411	Grade 12-24-12	58	0.03	0.004	0.008	1.30
170511	Grade 16-1-0	33	0.01	0.002	0.006	0.90
180311	Grade 3-1-2	33	0.01	0.002	0.006	0.66
170711	Grade 7-25-40	19	0.002	0.001	0.005	0.43
170211	Grade 27-0-0	20	0.002	0.001	0.005	0.40
171011	Epsom Salts	12	0.001	0.001	0.005	0.62
170611	Grade 2-15-15 w/PO3	23	0.001	0.001	0.005	0.40

IA ratio = Rob. SD/IA_{SD}

Red indicates IA Ratio significantly > 1

Plot of IA Ratio vs Concentration (mf on Log scale) For Different IA Equation Groups



Ideally these curves should be flat – 0 slope!

IA curves are systematically punitive at higher concentrations.



Micronutrient IA Calculation

IA is calculated from a Guarantee or Measured Value

$$\text{IA} = \text{Unit} + \% \text{ of Rob. Mean}$$

Analyte	Unit	%
Ca	0.2	5
Mg	0.2	5
S	0.2	5
B	0.003	15
Cu	0.005	10
Fe	0.005	10
Mn	0.005	10
Na	0.005	10
Zn	0.005	10
Co (ppm)	1	30
Mo (ppm)	1	30

$$\text{IA} = 0.2 + 0.05 \times \text{Rob. Mean}$$

$$\text{IA} = 0.003 + 0.15 \times \text{Rob. Mean}$$

Not many data points but revealing.

$$\text{IA} = 0.005 + 0.1 \times \text{Rob. Mean}$$

$$\text{IA} = 1 + 0.3 \times \text{Rob. Mean}$$

A series of Linear Equations.

**AND THIS IS WHERE WE GO
DOWN THE RABBIT HOLE!**





Horwitz Approach, J. AOAC, 1980

Relationship between σ (Reproducibility SD) and concentration C (mass fraction).

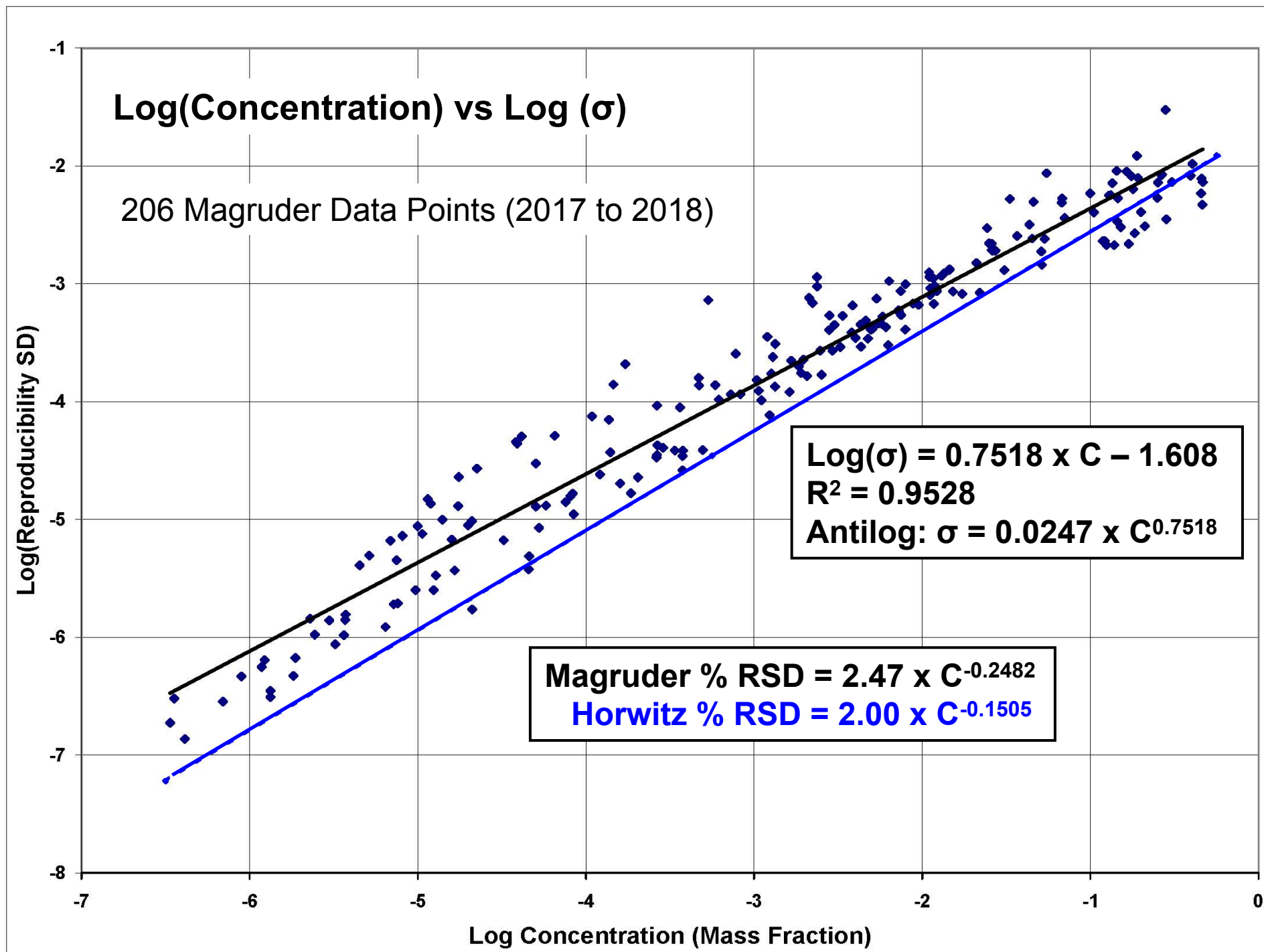
$$\sigma = AC^B \quad \text{Where A and B are constants}$$

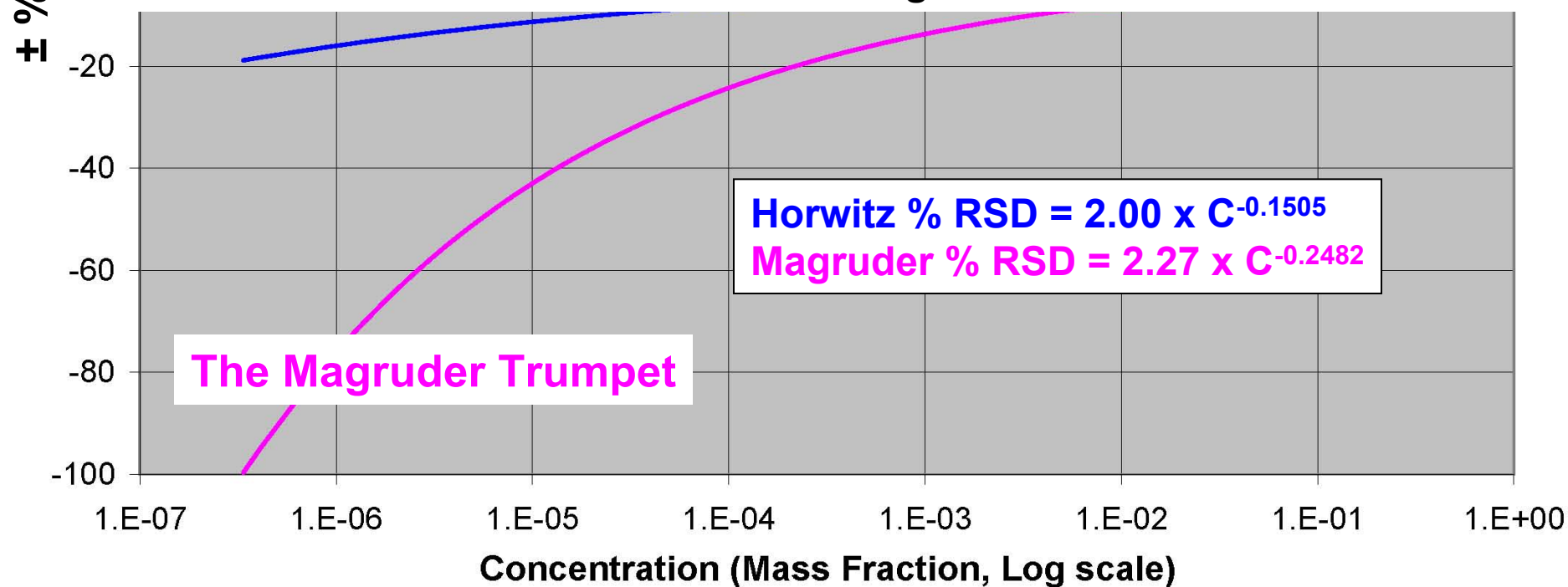
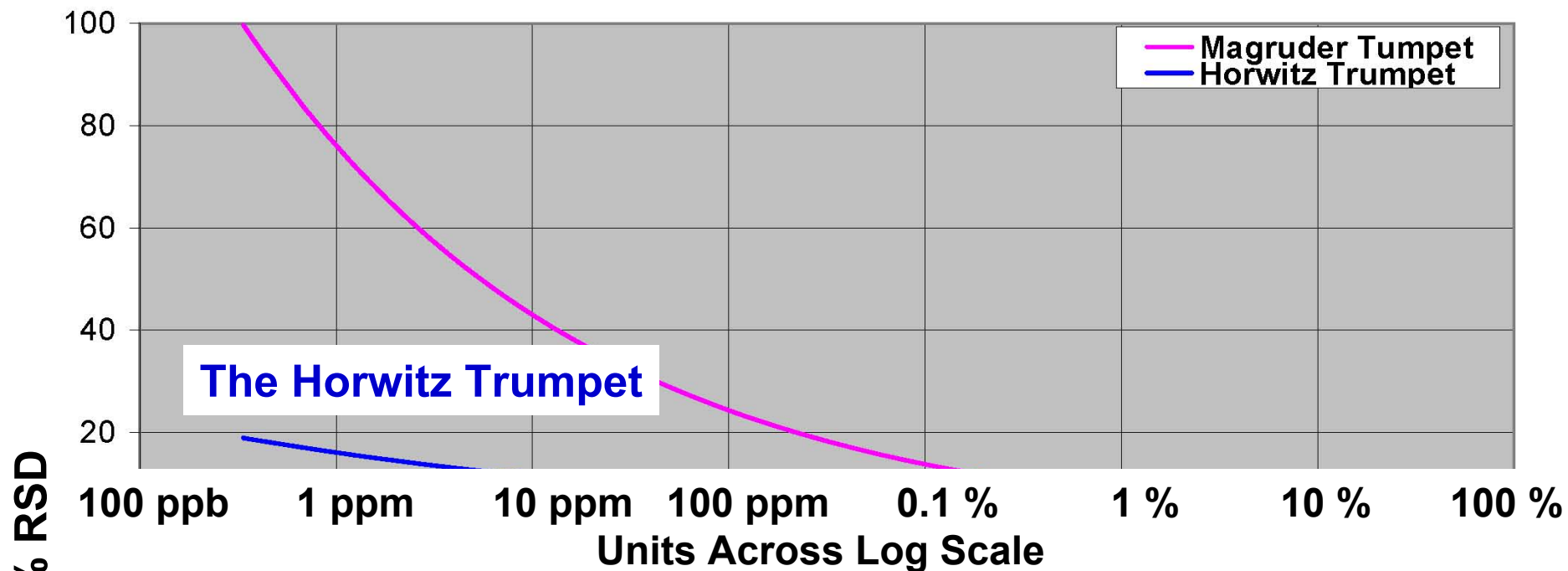
$$\text{Log}(\sigma) = \text{Log}(A) + B \times \text{Log}(C)$$

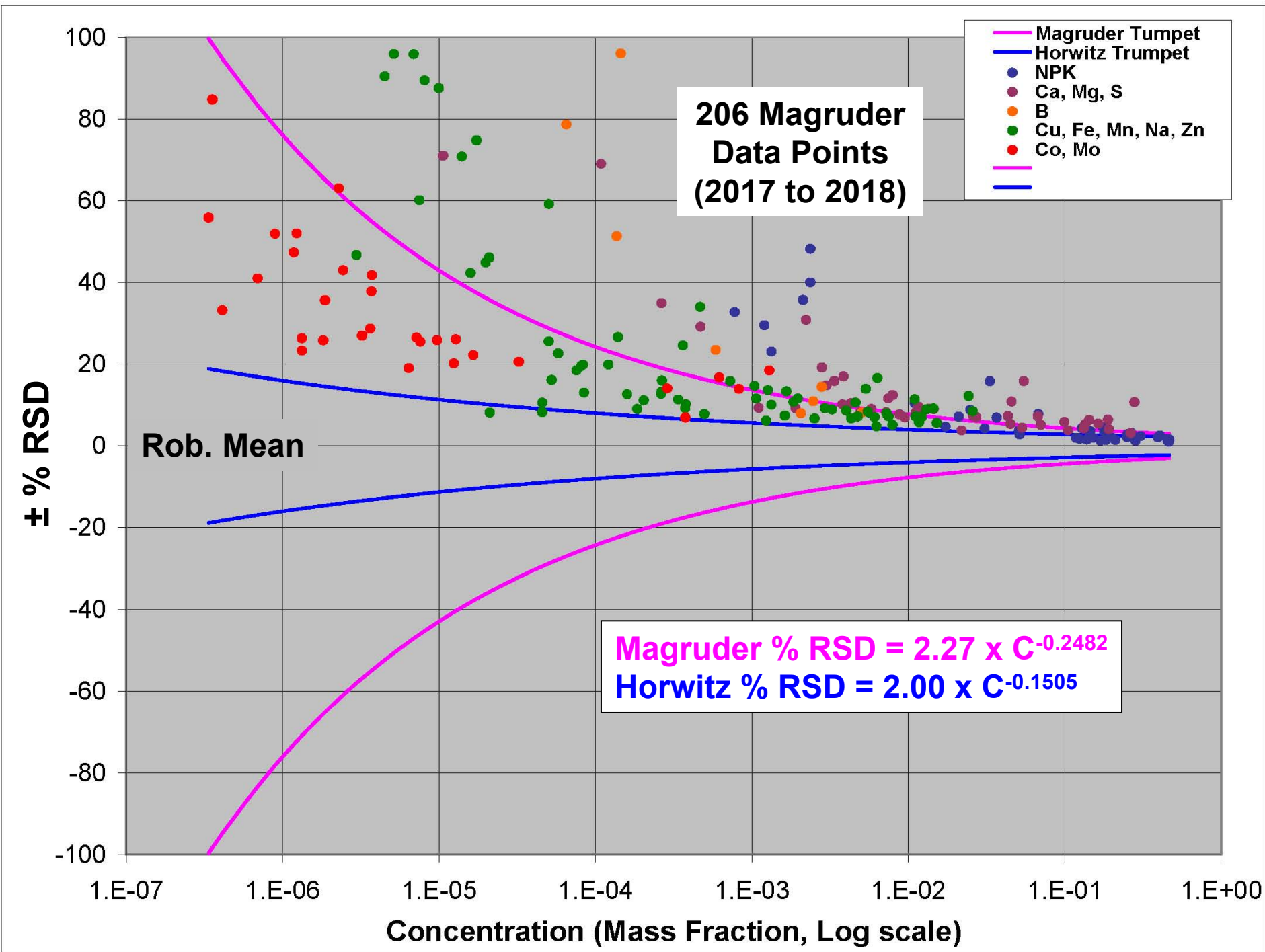
Straight line plot of Log (reproducibility SD) vs Log (concentration)
With Slope B and Intercept Log(A).

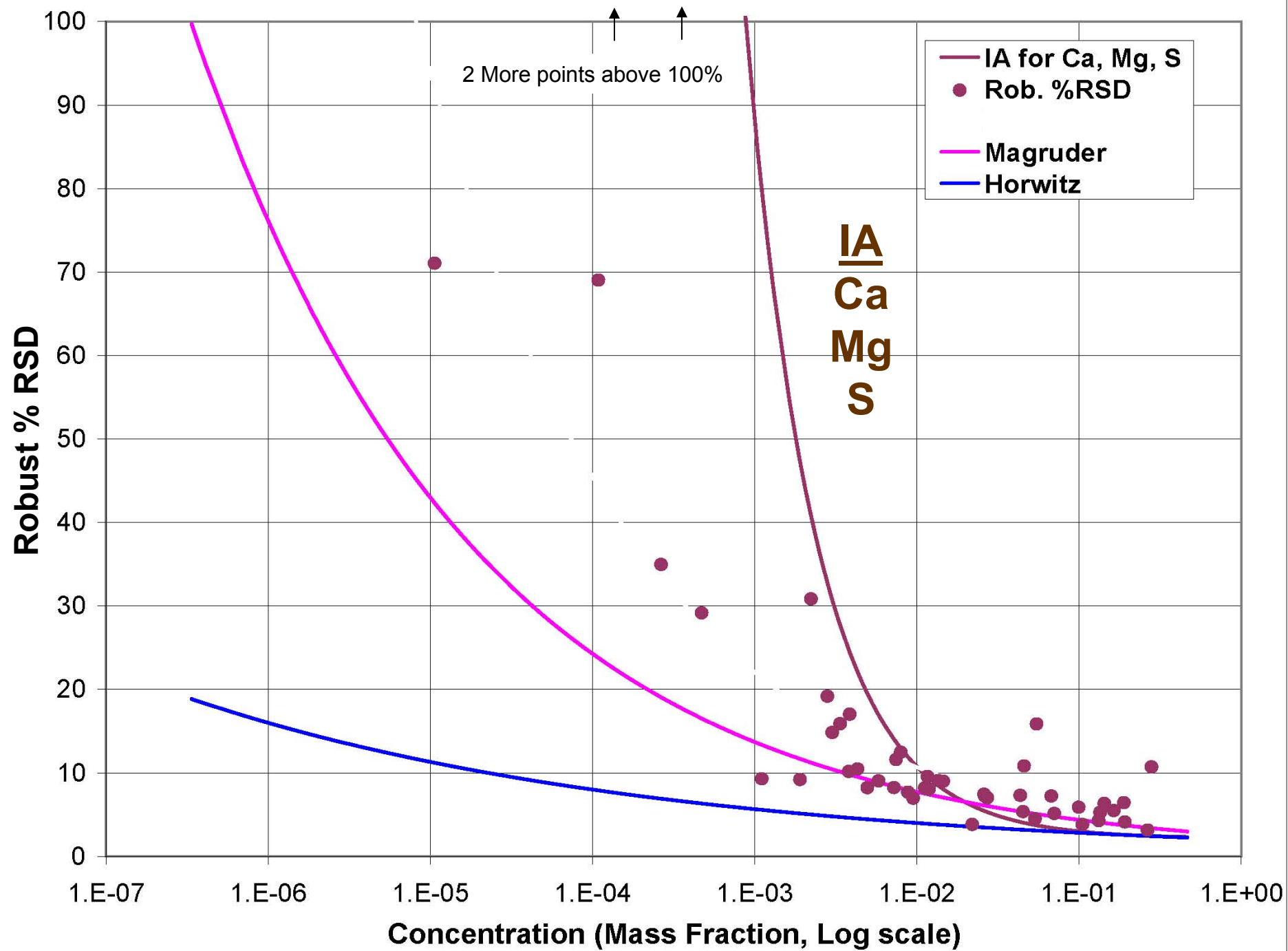
Now we apply this logic to the Magruder Rob. Mean and Rob. SD
For 206 Analytes from 2017 to current and plot this curve:

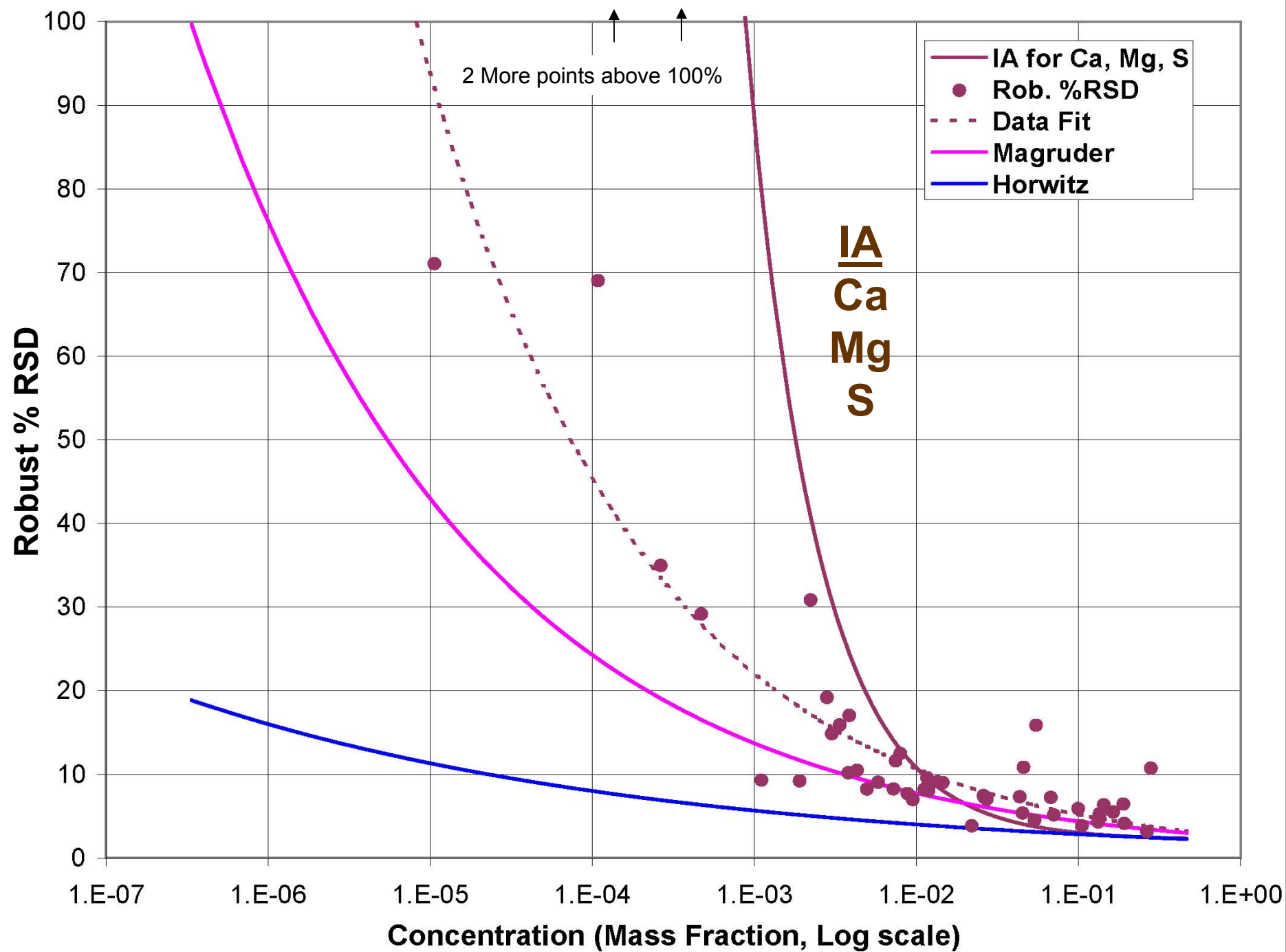
$$\text{Log}(\text{Rob. SD}) = \text{Log}(A) + B \times \text{Log}(\text{Rob. Mean as mass fraction})$$

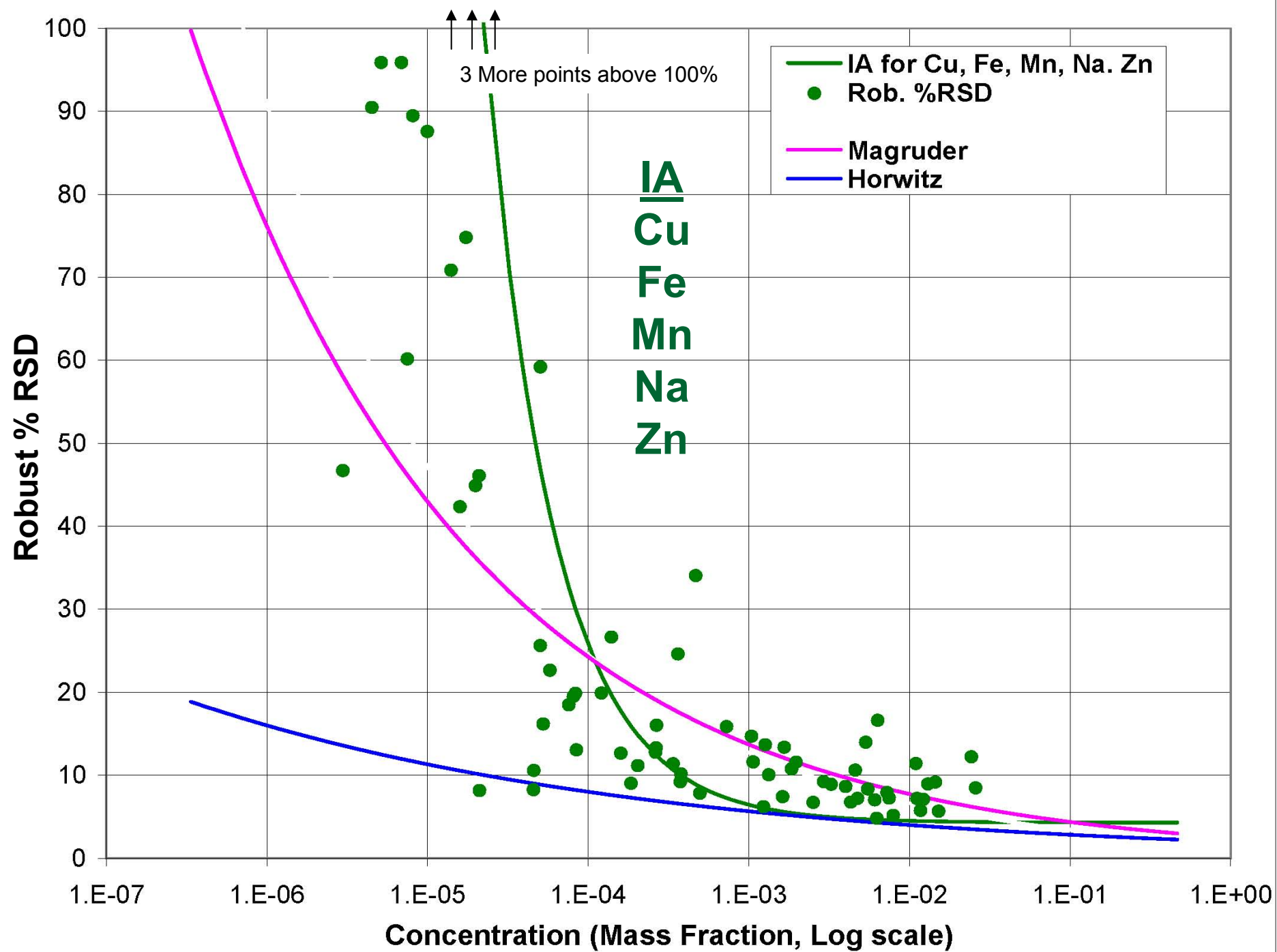


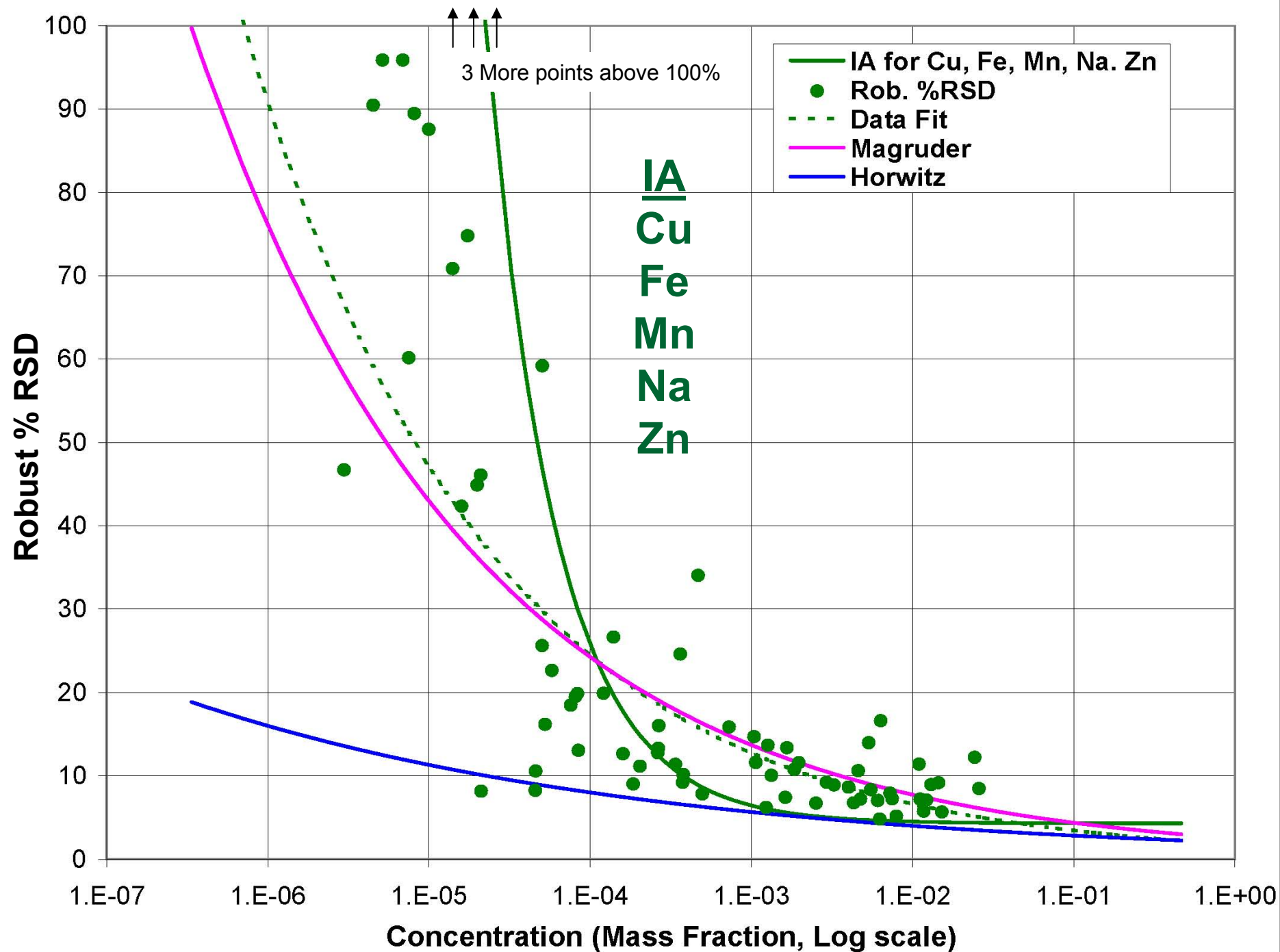


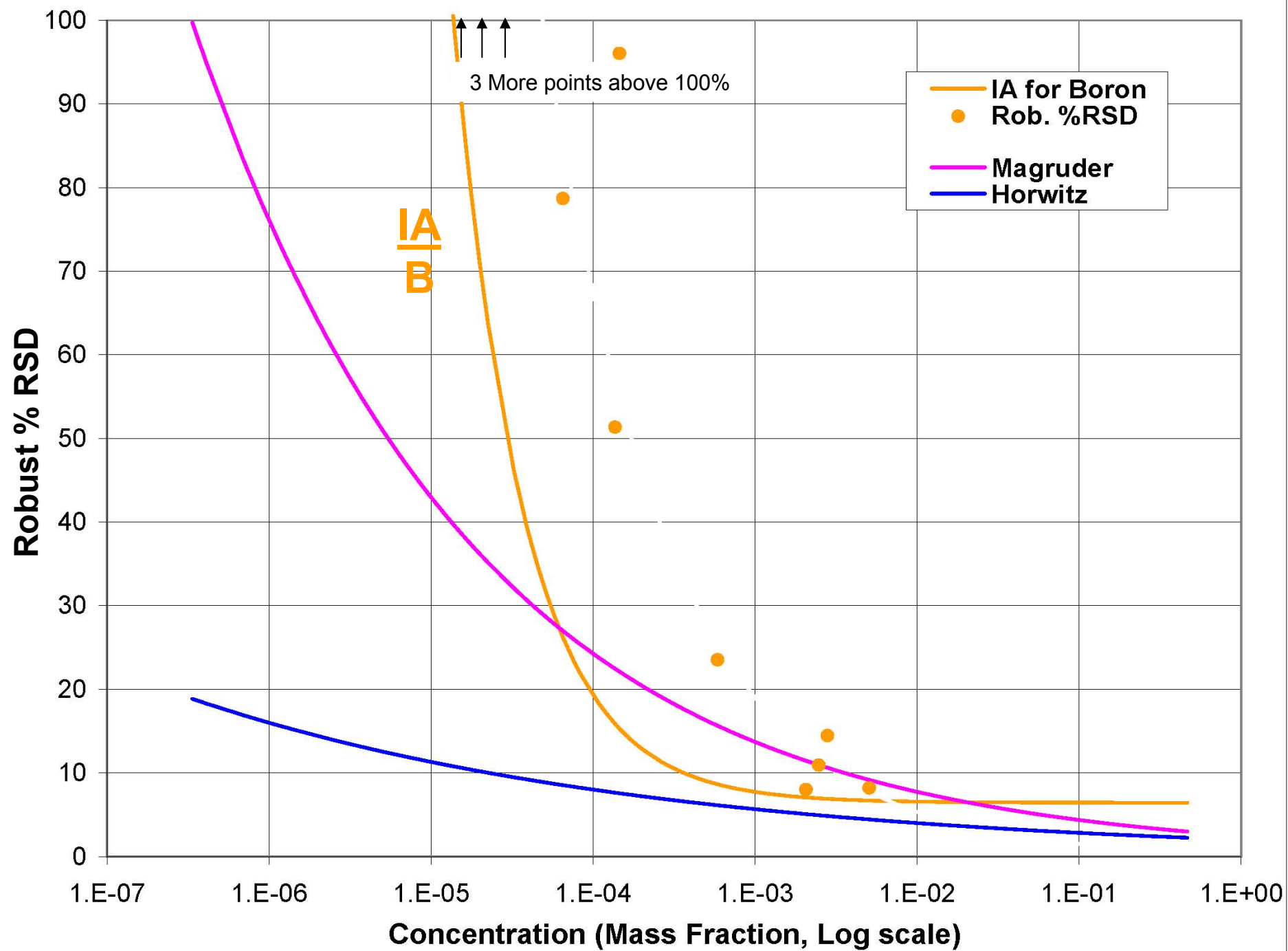


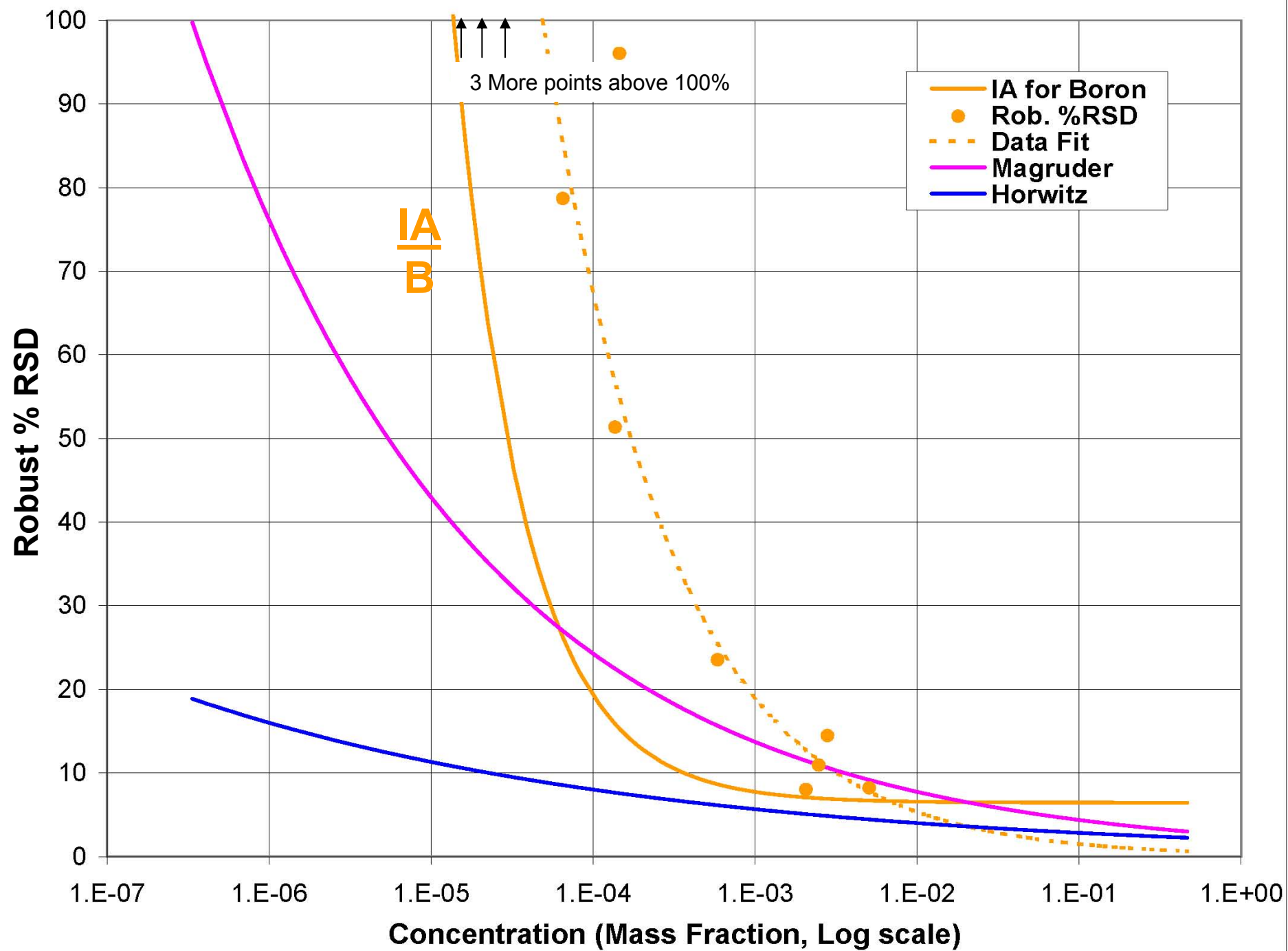


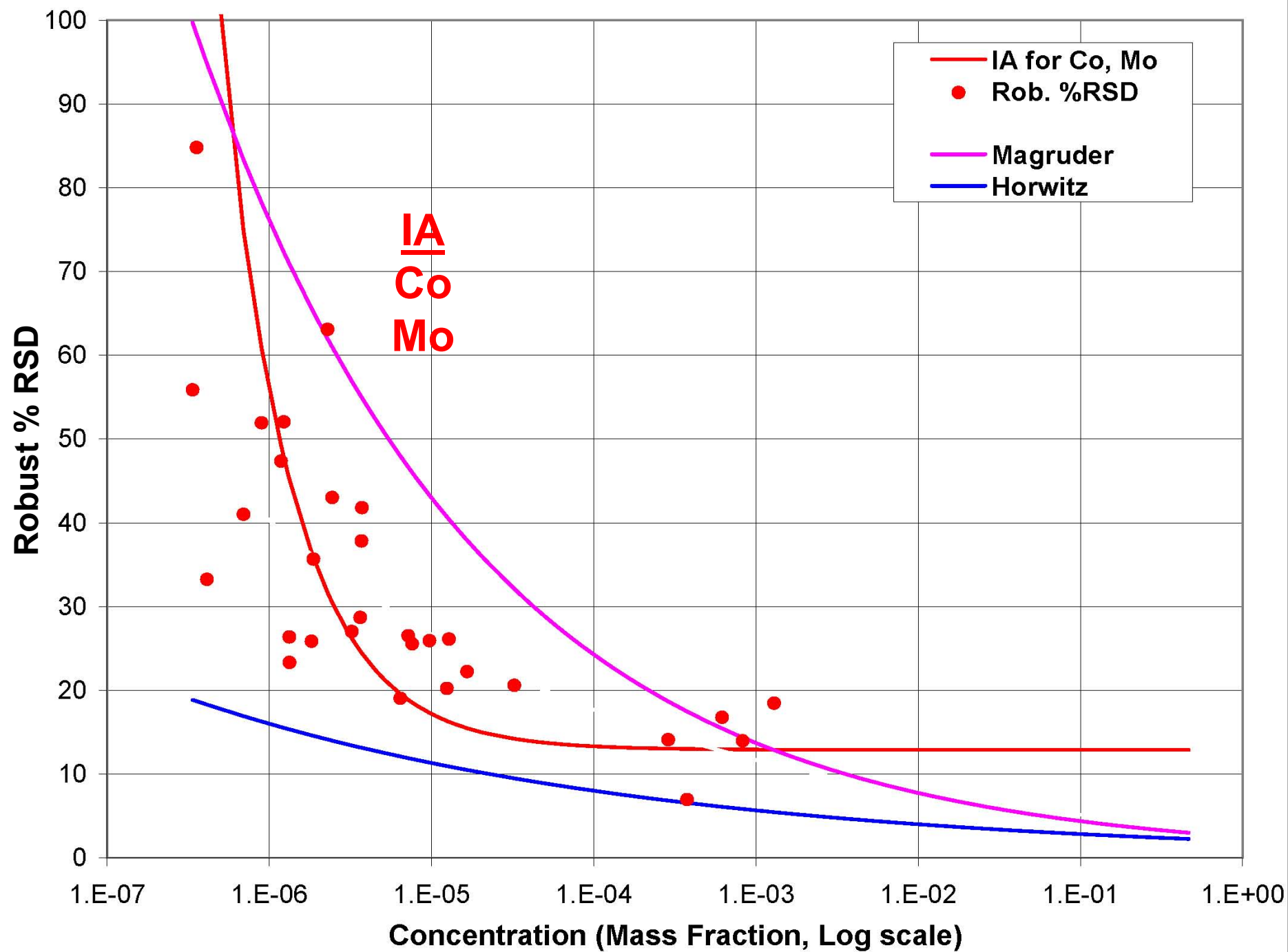


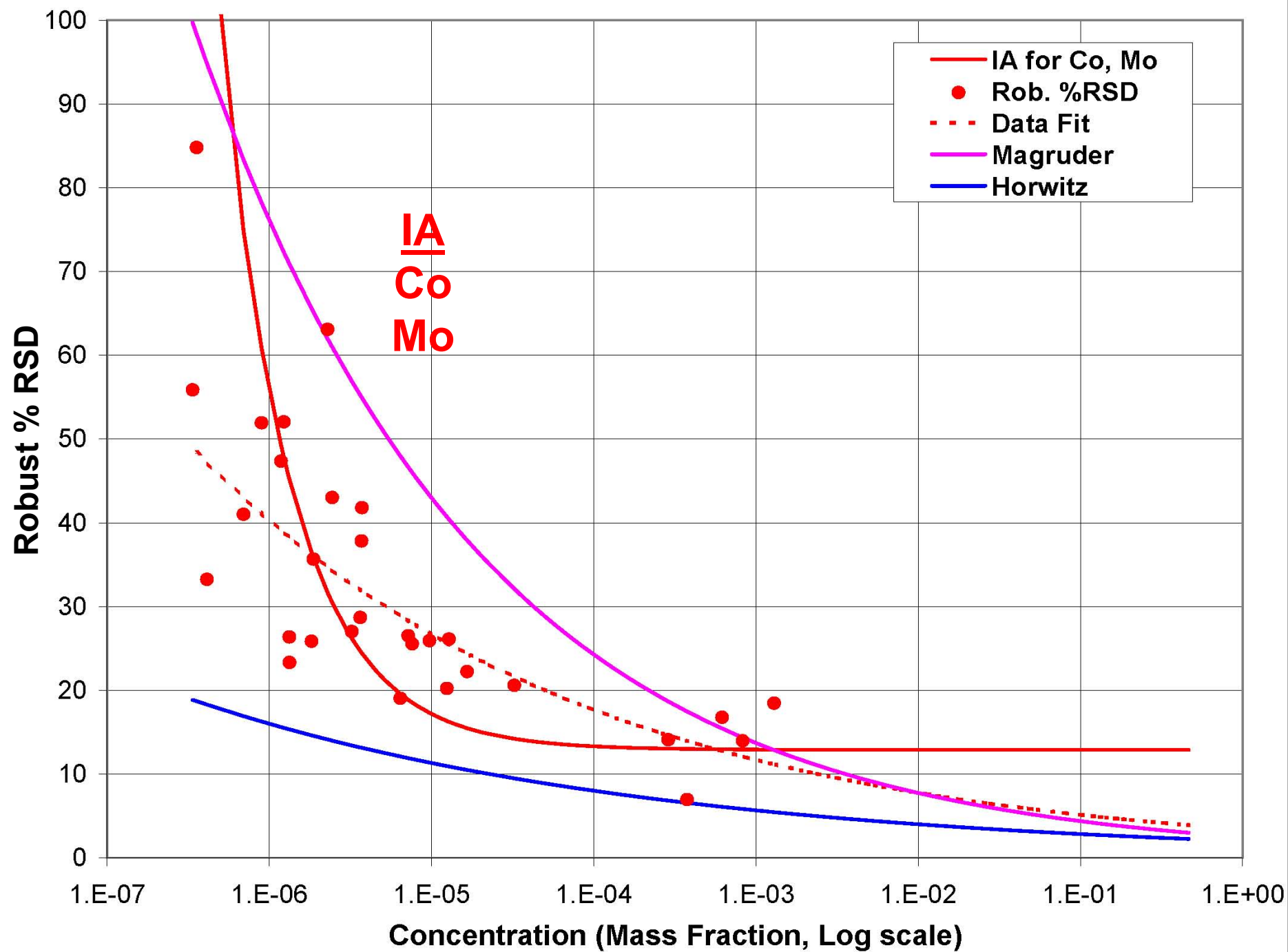












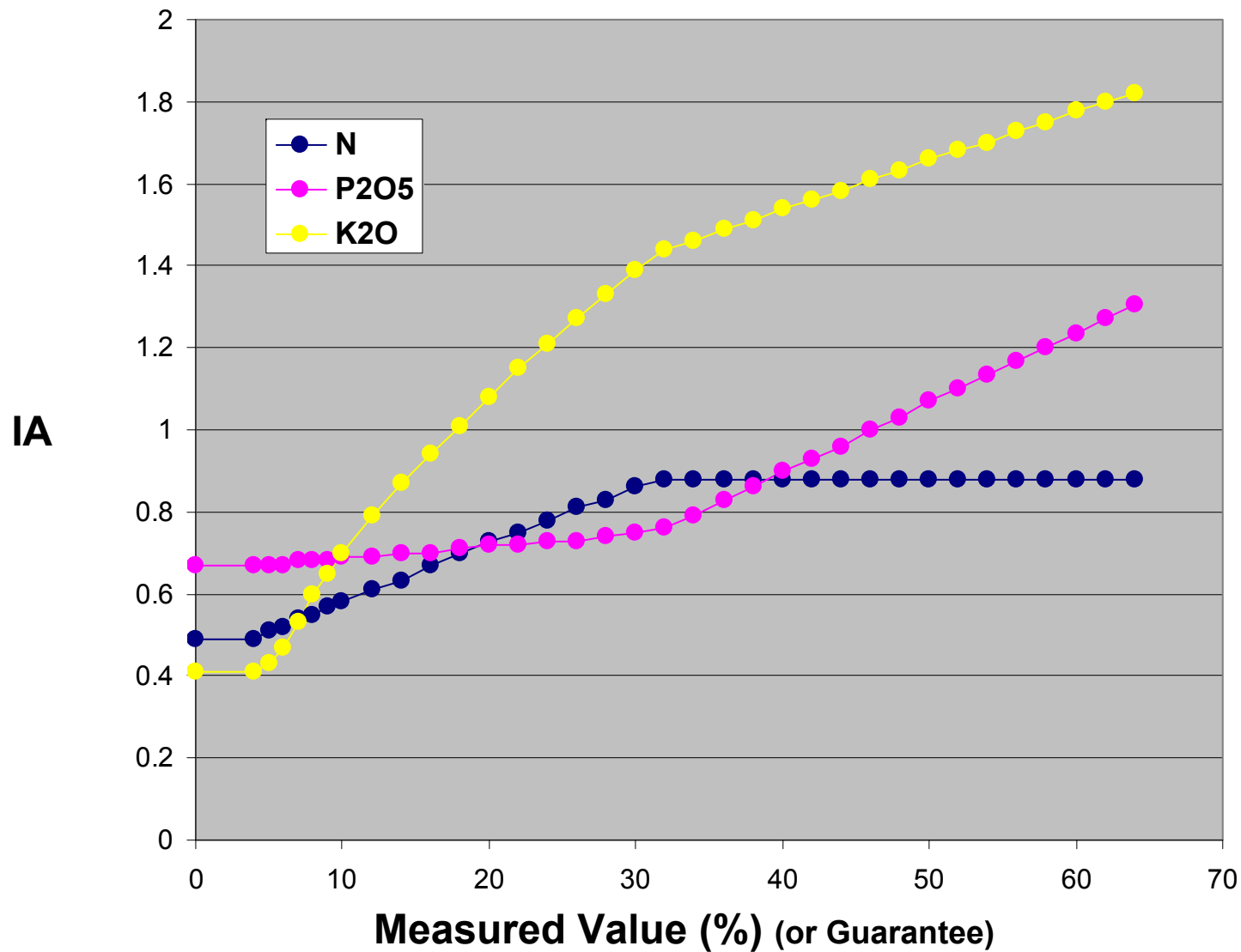
Review of Equations

Analyte	IA from OP % * AV + Unit	Data Fit to $A * C^B$
Ca, Mg, S	$0.05 * AV + 0.02$	$0.025 * C^{0.6854}$
Cu, Fe, Mn, Na, Zn	$0.10 * AV + 0.005$	$0.018 * C^{0.7158}$
Co, Mo	$0.3 * AV + 1$	$0.034 * C^{0.8210}$
Overall Magruder Curve	na	$0.025 * C^{0.7518}$

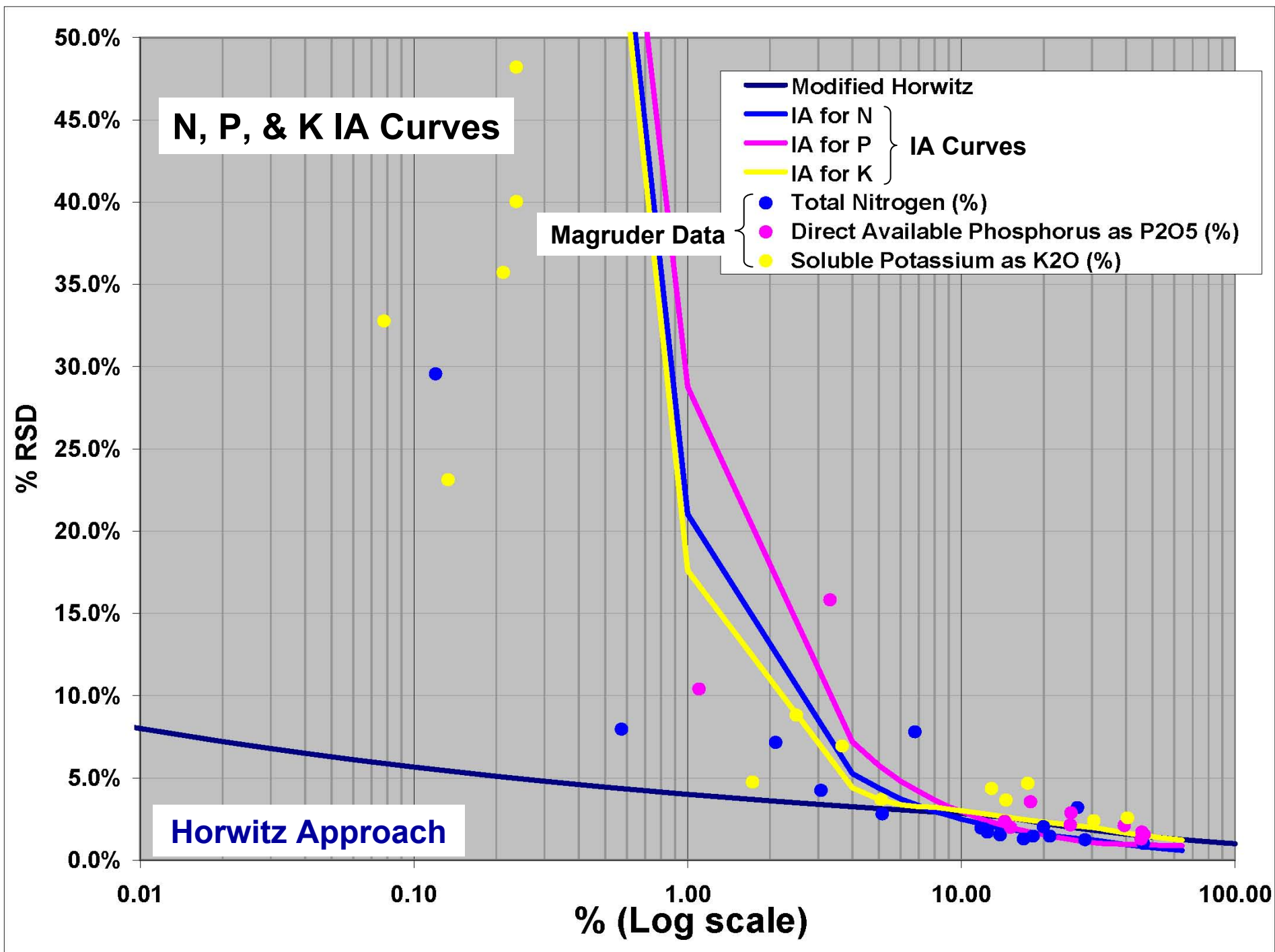
AV = Assigned Value or Rob. Mean.
C = Assigned Value as mass fraction.

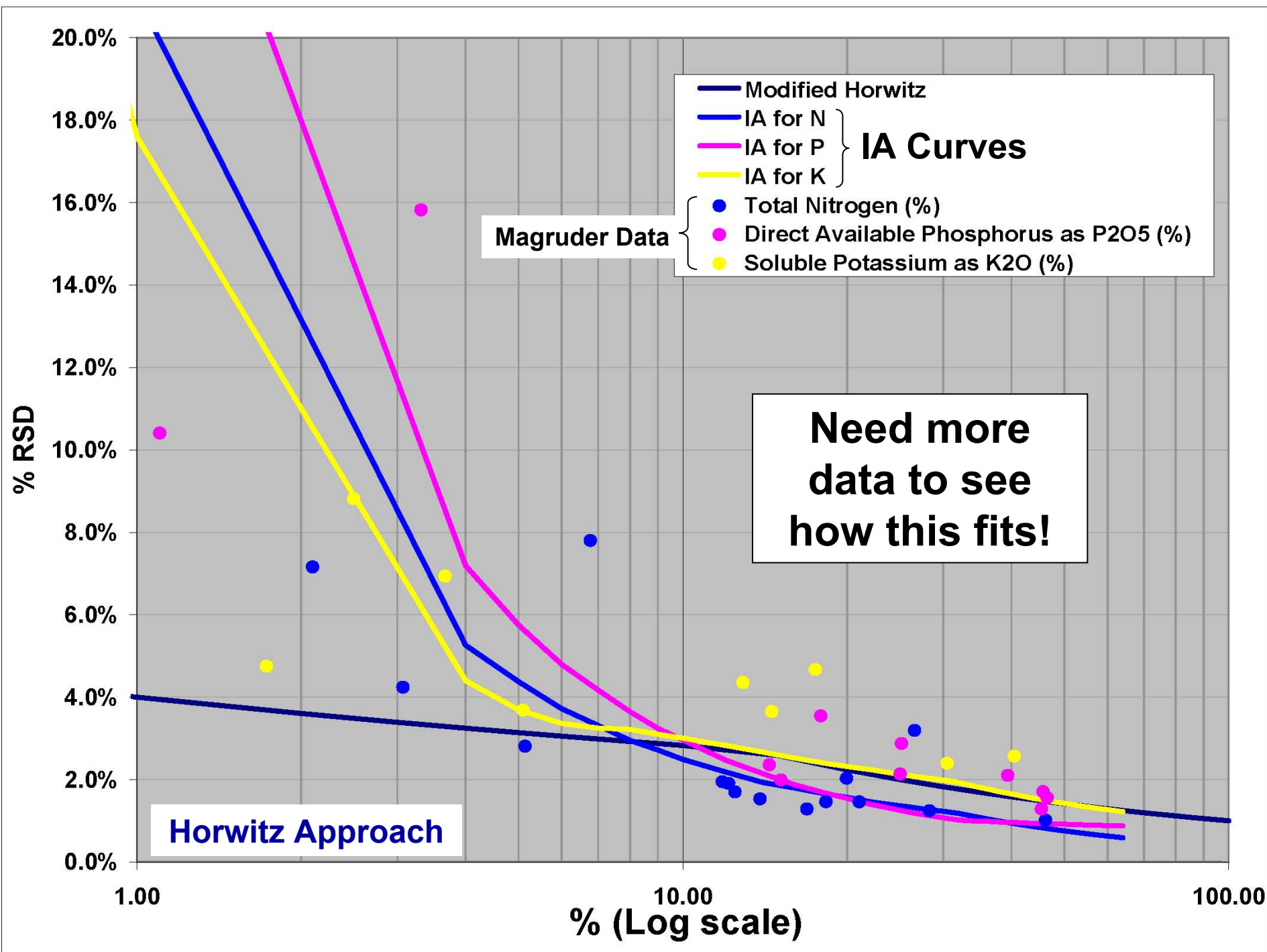


N-P-K IA Table Values (Plotted from OP)

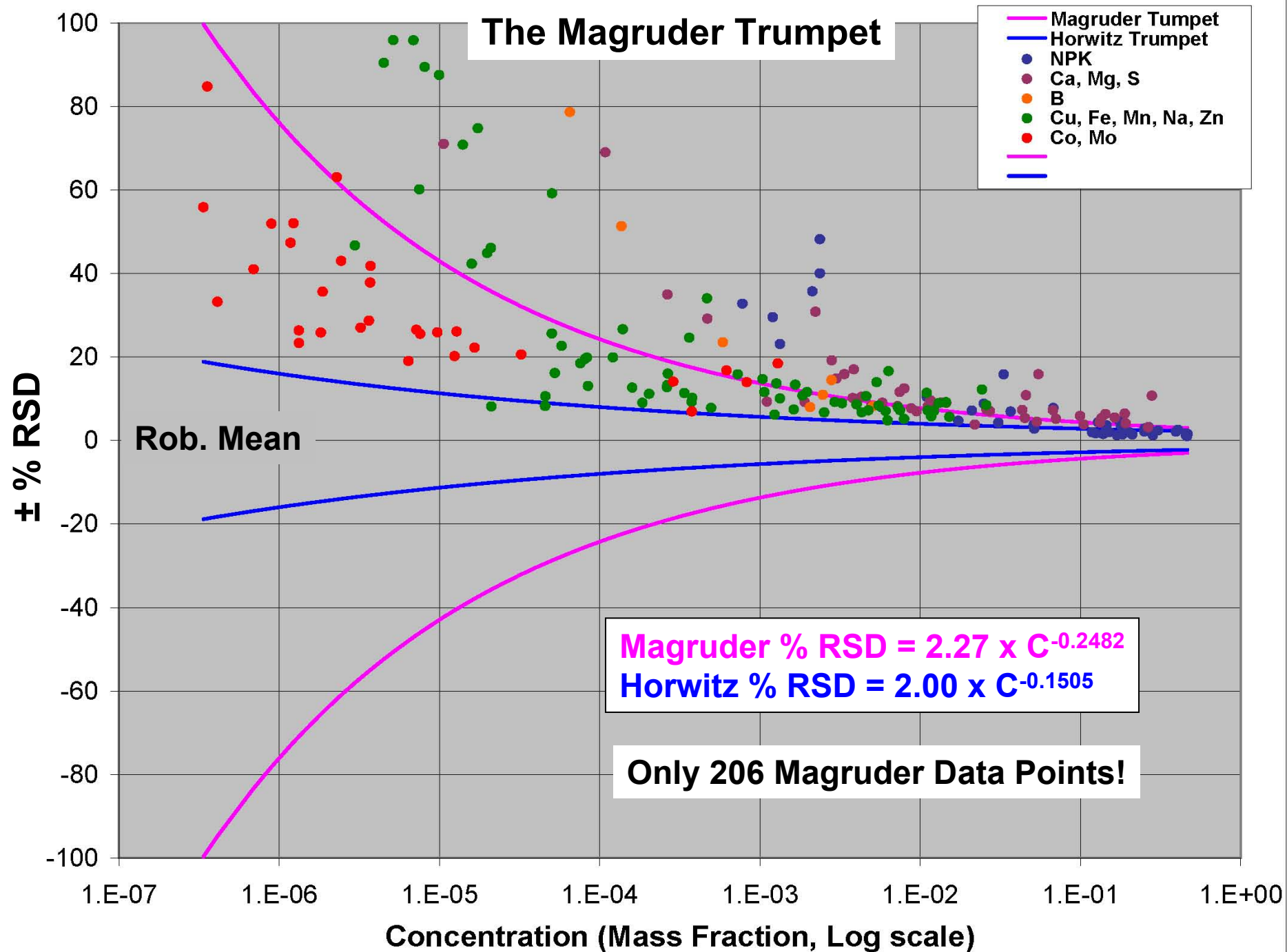


From: AAPFCO OP





The Magruder Trumpet



Some Considerations

- In time with more data we can refine the Magruder curve.
- Careful parsing of aberrant data points should make the Magruder Trumpet less like a French Horn.
- The Horwitz curve could provide a useful baseline.



In This Discussion

- Unless pointed out, all reference to IA is to IA_{SD} .
 - This describes 68% of the data.
 - It is more common to allow 95% or 99%.
 - This can be achieved by expanding the IA_{SD} observed or calculated, using Normal curve probability factors – one-tail or two-tail depending on need.
-

Summary and Conclusions

- Current IA calculations from the OP tend to be more relaxed at lower concentrations and more punitive at higher concentrations.
 - The IA Ratio is the first indicator that the OP IA equations may **not** represent the current status of inter laboratory variance.
 - There appears to be a strong Log-Log relationship between the Repeatability SD and Concentration across 7 orders of magnitude.
 - The NPK IA equations contain non-linear subtleties. We will need a lot more data to see this.
 - As we accumulate more data, the Horwitz like approach may be a way to reevaluate IAs. We do need more data to rule out anomalies hampering the data fit.
-



Questions?